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Holographic approach to a minimal Higgsless model

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Abstract: In this work, following an holographic approach, we carry out a low energy effective study of a minimal Higgsless model based on SU(2) bulk symmetry broken by boundary conditions, both in flat and warped metric. The holographic procedure turns out to be an useful computation technique to achieve an effective four dimensional formulation of the model taking into account the corrections coming from the extra dimensional sector. This technique is used to compute both oblique and direct contributions to the electroweak parameters in presence of fermions delocalized along the fifth dimension.

Keywords: Beyond Standard Model, Large Extra Dimensions.

Contents

1. Introduction

A relevant issue in the context of high energy physics is that extra dimensions provide alternative ways for breaking gauge symmetries with respect to the famous Higgs mechanism, [1 − 5]. Furthermore, an additional non trivial feature of the Yang-Mills theories in a compact extra dimension is that the WW and ZZ elastic scattering amplitudes can be unitarized by the tower of heavy gauge modes, [6 − 9], and the unitarity of the theory is postponed to a higher scale. This additional scale is related to the fact that the theory becomes strongly interacting and the number of modes that can contribute to the amplitudes at high energy grows as well. Thus extra dimensional models provide alternative methods with respect to the standard theory to break the gauge symmetries giving mass to the gauge bosons and preserve the unitarity of the W and Z scattering at high energy. Since the spontaneous electroweak symmetry breaking via the Higgs mechanism and the unitarity restoration via the Higgs boson exchange are the main arguments for the existence of the Higgs boson, then the Higgs sector can be eliminated in favor of a compact extra dimensional sector. The class of models, usually formulated in five dimensions based on this assumption is named Higgsless [8, 10 − 17].

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Extra dimensional extensions of the Standard Model (SM) share some similarities with strongly interacting models at effective level as can be inferred through the AdS/CFT correspondence [18]. The analogy between the technicolor models and extra dimensional models arises also by discretizing the extra dimensional theory with the dimensional deconstruction mechanism. In fact the deconstruction mechanism provides a correspondence at low energies between theories with replicated 4D gauge symmetries G and theories with a 5D gauge symmetry G on a lattice [19, 25]. We will refer to the models with replicated gauge symmetries as moose models [26 – 30].

Since in the original versions of Higgsless models the fermions coupled only with the standard gauge fields, the electroweak parameters $\epsilon_1, \epsilon_2, \epsilon_3$ or S, T, U, [31 – 33], had only oblique contributions. These oblique corrections tend to give large and positive contributions to the ϵ_3 (or S) parameter, so that it is difficult to conciliate the electroweak bounds with a delay of the unitarity violation scale, [12, 30]. However a delocalization of the fermionic fields into the bulk as in [34, 35], that is allowing standard fermions with direct coupling to all the moose gauge fields as in [36], leads direct contributions to the electroweak parameters that can correct the bad behavior of the ϵ_3 parameter.

The fine tuning which cancels out the oblique and direct contributions to ϵ_3 independently in each bulk point, that is from each internal moose gauge group, corresponds to the so called ideal delocalisation of the fermions, [36 – 38]. Therefore, it is worthwhile investigating whether it is possible to reproduce such an ideal delocalisation starting from the theoretical assumption of a 5D Dirac sector with an appropriate choice of the boundary conditions on the branes.

The determination of the low energy observables in extra dimensional models is generally achieved through a recursive elimination of the heavy Kaluza Klein (KK) excitations from the equations of motion. However, a much more useful way to reach the effective theory is described in [39, 12, 13]. This method is broadly inspired by the holographic technique which allows the reduction of the 5D theory into a four dimensional one [40, 43].

Here we will study the problem of the ϵ_3 fine tuning directly in the extra dimensional formulation of the Higgsless model using the holography as a powerful procedure of calculus. In fact the bulk physics can be taken into account by solving the bulk equations of motion with given boundary conditions, so that one is left with a boundary or holographic action, which is, indeed, a 4D theory related to the original extra dimensional one.

Other solutions to get a suppressed contribution to ϵ_3 have been investigated like the one suggested by holographic QCD, assuming that different five dimensional metrics are felt by the axial and vector states [44, 47]. However recently it has been shown that the backgrounds that allow to get a negative oblique contribution to ϵ_3 are pathological, since require unphysical Higgs profile or higher dimensional operators [48].

In this note we consider a 5D version of a linear moose model previously proposed, [28, 36, 49]. The right pattern for electroweak symmetry breaking is obtained by adding to the SU(2) five dimensional gauge symmetry, extra terms on the branes and boundary conditions breaking the symmetry to the $\mathrm{U}(1)_{\mathrm{em}}$. Then we evaluate the oblique corrections through the vacuum amplitudes of the standard gauge bosons which can be easily obtained from the holographic formulation. In general, the results obtained with the holographic procedure are in agreement with the continuum limit of the corresponding linear moose model studied in [36, 49]. In presence of direct couplings of the bulk gauge fields to standard fermions, effective fermion current-current interactions, which are obtained in the deconstruction analysis [36], are recovered from the full effective action solving the complete bulk equations of motion with a suitable perturbative expansion [39].

In section 2 we review the holographic description of the gauge sector and show how to derive the oblique contributions to the electroweak parameters. In section 3 we perform a holographic analysis of the fermions in 5D by solving the equations of motion with suitable boundary conditions and project out the bulk dynamics on the branes, as in [50]. In section 4 we use the bulk solutions obtained for the gauge and fermion fields in the interaction terms, and we derive the low energy effective action from which we compute the ϵ parameters. The results obtained in the flat scenario are then extended to the warped background in section 5. Conclusions are given in section 6.

2. Holographic analysis of the gauge sector

We review in this section the continuum limit of the moose model of [26, 28, 35, 36, 51, 49] and the holographic approach for gauge fields proposed in [12].

We start from an action based on a 5D Yang-Mills theory defined on a segment, with SU(2) bulk gauge symmetry and flat metric:

$$S_{\rm YM}^{\rm bulk} = -\frac{1}{2g_5^2}\int d^4x \int_0^{\pi R} dy \mathsf{Tr}[F^{MN}(x,y)F_{MN}(x,y)]\,, \tag{2.1}$$

where g_5 is the bulk gauge coupling with mass dimension $-1/2$, $F^{MN} = F^{aMN}T^a$, being T^a the generators of the SU(2) symmetry such that $\mathsf{Tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$, $T^a = \frac{\tau^a}{2}$ where τ^a are the Pauli matrices and $F(A)^{aMN} = A^{aMN} + i\epsilon^{abc}A_b^M A_c^N$ with $A^a_{MN} = \partial_M A^a_N - \partial_N A^a_M$.

We will work in the unitary gauge $A_5^a \equiv 0$, which gives useful simplifications. Then integrating by parts the bulk action in eq. (2.1) and neglecting the trilinear and quadrilinear couplings coming from the non-abelian terms of the field strength, we are lead to a bilinear action written as a boundary term plus a bulk term [12].

From the bulk term we can get the bulk equations of motion which, in the four momentum space, for the transverse and longitudinal components of the gauge field, are respectively ($\partial_5 = \partial_y$):

$$(\partial_5^2 + p^2)A^t_\mu(p,y) = 0, \qquad\qquad \partial_5^2 A^l_\mu(p,y) = 0\,. \tag{2.2}$$

Furthermore, as long as one considers processes with all external particles with mass m_f much lighter than the gauge vector mass m_A, the longitudinal part of the two point function yields a suppression of the order $(m_f/m_A)^2$. Thus, in discussing the electroweak corrections coming from the extra gauge sector, we will investigate only the transverse components of the gauge field, [52] (for sake of simplicity from now on we will omit the superscript).

Let us impose the first of the eqs. (2.2) as a constraint, therefore the residual bilinear part of the 5D action is an holographic term, [12],

$$\mathcal{S}_{\rm YM}^{(2)holog} = -\frac{1}{g_5^2}\int \frac{d^4p}{(2\pi)^4} Tr[A_\mu(p,y)\partial_5 A^\mu(p,y)]_0^{\pi R}\,. \tag{2.3}$$

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As said, in addition to this bilinear boundary action obtained imposing the linear equations of motion eqs. (2.2), we have the trilinear and quadrilinear bulk terms coming from the non-abelian part of the 5D SU(2) Yang-Mills theory. Anyway they are not involved, at the leading order, in the electroweak parameter tree level estimation and have weaker experimental bounds [53].

In order to solve the bulk equations of motion (2.2) we need to assign boundary conditions for each bulk field component. We will fix these conditions by requiring to recover the SM gauge content at the extremes of the segment (branes). In order to get this, following [12], we add, besides the brane kinetic terms, mass brane terms for the gauge fields:

$$\begin{aligned} S_{\rm YM}^{\rm brane} = &-\frac{1}{2\tilde{g}^2}\int d^4x\int_0^{\pi R} dy\delta(y)\mathsf{Tr}[F(A)^{\mu\nu}(x,y)F(A)_{\mu\nu}(x,y)] \\ &-\frac{1}{4\tilde{g}'^2}\int d^4x\int_0^{\pi R} dy\delta(y-\pi R)F^{3\mu\nu}(x,y)F^3_{\mu\nu}(x,y) \\ &+\frac{c_1^2}{2\tilde{g}^2}\int d^4x\int_0^{\pi R} dy\delta(y)Tr[(A^\mu-\tilde{g}\tilde{W}^\mu)(A^\mu-\tilde{g}\tilde{W}_\mu)] \\ &+\frac{c_2^2}{4\tilde{g}'^2}\int d^4x\int_0^{\pi R} dy\delta(y-\pi R)[(A^{3\mu}-\tilde{g}'\tilde{\mathcal{Y}}^\mu)(A^3_\mu-\tilde{g}'\tilde{\mathcal{Y}}_\mu)+A^{1,2\mu}A^{1,2}_\mu]\,, \end{aligned} \tag{2.4}$$

The parameters $c_{1,2}$ have the dimension of a 4D mass and in the limit $c_{1,2}\to\infty$ fix the boundary values of the bulk field to the standard gauge fields $\tilde{W}_\mu = \tilde{W}^a_\mu T^a$ and $\tilde{\mathcal{Y}}_\mu$ (the tilde indicates unrenormalized quantities):

$$\begin{aligned} A^\pm_\mu(x,y)|_{y=0} &\equiv \tilde{g}\tilde{W}^\pm_\mu(x), \qquad & A^\pm_\mu(x,y)|_{y=\pi R} &\equiv 0, \\ A^3_\mu(x,y)|_{y=0} &\equiv \tilde{g}\tilde{W}^3_\mu(x), \qquad & A^3_\mu(x,y)|_{y=\pi R} &\equiv \tilde{g}'\tilde{\mathcal{Y}}_\mu(x)\,. \end{aligned} \tag{2.5}$$

In this way the standard fields are introduced as auxiliary fields. Though we are considering the flat metric case, these fields are the analogous of the source fields of the AdS/CFT correspondence. Indeed, we are imposing standard gauge symmetry $\mathrm{SU}(2)_L$ on the $y=0$ brane, and $\mathrm{U}(1)_Y$ on the $y=\pi R$ one.

We are now able to write down the holographic formulation of the model by imposing the bulk equations of motion given in eqs. (2.2) and the boundary conditions (2.5). The resulting Lagrangian density in momentum space at quadratic level is

$$\begin{aligned} \mathcal{L}_{\rm YM}^{(2)holog+SM} = &-\frac{\tilde{g}'}{2g_5^2}\tilde{\mathcal{Y}}^\mu(p)\partial_y A^3_\mu(p,y)|_{y=\pi R}+\frac{\tilde{g}}{2g_5^2}\tilde{W}^{a\mu}(p)\partial_y A^a_\mu(p,y)|_{y=0} \\ &+\frac{p^2}{2}\tilde{W}^a_\mu(p)\tilde{W}^{a\mu}(p)+\frac{p^2}{2}\tilde{\mathcal{Y}}_\mu(p)\tilde{\mathcal{Y}}^\mu(p)\,. \end{aligned} \tag{2.6}$$

Let us comment on the relation between this holographic approach and the one in which one uses the KK expansion in normal modes for the gauge field. In this latter case the boundary conditions to be imposed are different as can be derived by varying the bulk action with brane kinetic terms added, [54, 55]. It can be shown that, fixing the bulk field on the boundary according to eqs. (2.5) is indeed coherent with an effective description in

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terms of Dirichlet and Neumann boundary conditions obtained from the variation of an extra dimensional theory based on the bulk action with brane kinetic terms. By expanding the bulk fields in terms of KK eigenfunctions

$$A^a_\mu(p,y) = \sum_n f^a_n(y) A^{a(n)}_\mu(p)\,, \tag{2.7}$$

after imposing Neumann conditions on both the branes for the neutral component of the bulk field as well as Neumann condition on the $y=0$ brane and Dirichlet condition on $y=\pi R$ brane for the charged components of the bulk field [26], at leading order in $\tilde{g}^2\pi R/g_5^2$ (in such a way that the heavy non standard KK mass eigenstates can be neglected), we obtain

$$\begin{aligned}
A^\pm_\mu(p,0) &\sim f^\pm_0(0) W^\pm_\mu(p) \sim \frac{\tilde{e}}{s_{\tilde{\theta}}}\tilde{W}^\pm_\mu(p),\\
A^\pm_\mu(p,\pi R) &\sim f^\pm_0(\pi R) W^\pm_\mu(p) \equiv 0,\\
A^3_\mu(p,0) &\sim f^3_0(0)A_\mu(p) + f^3_1(0)Z_\mu(p) \sim \tilde{e}\tilde{A}_\mu(p) + \tilde{e}\frac{c_{\tilde{\theta}}}{s_{\tilde{\theta}}}\tilde{Z}_\mu(p) = \tilde{g}\tilde{W}^3_\mu(p)\,,\\
A^3_\mu(p,\pi R) &\sim f^3_0(\pi R)A_\mu(p) + f^3_1(\pi R)Z_\mu(p) \sim \tilde{e}\tilde{A}_\mu(p) - \tilde{e}\frac{s_{\tilde{\theta}}}{c_{\tilde{\theta}}}\tilde{Z}_\mu(p) = \tilde{g}'\tilde{\mathcal{Y}}_\mu(p)\,,
\end{aligned} \tag{2.8}$$

where we have introduced the SM neutral fields through the standard rotation: $\tilde{W}^3_\mu = c_{\tilde{\theta}}\tilde{Z}_\mu + s_{\tilde{\theta}}\tilde{A}_\mu$, $\tilde{\mathcal{Y}}_\mu = -s_{\tilde{\theta}}\tilde{Z}_\mu + c_{\tilde{\theta}}\tilde{A}_\mu$ with $\tilde{e} = \tilde{g}s_{\tilde{\theta}}$ and $t_{\tilde{\theta}} = \tan\tilde{\theta} = \tilde{g}'/\tilde{g}$ and, again, the tildes are for unrenormalized quantities.

We see that the boundary values for the bulk field expressed in terms of the lowest lying KK modes in eqs. (2.8) correspond, at effective level, to the boundary conditions given in eqs. (2.5). In other words, the effective holographic Lagrangian for the boundary fields $\tilde{W}_\mu$ and $\tilde{\mathcal{Y}}_\mu$, which are not the mass eigenstates but linear combinations of all the KK modes, can be used to describe the lightest states of the KK tower in the limit of heavy mass of the KK excitations.

2.1 Precision electroweak parameters

Let us now start the evaluation of the oblique corrections at tree level to the SM precision electroweak parameters by using the holographic Lagrangian density given in eq. (2.6).

Writing the generic solutions of the bulk equations of motion in terms of the interpolating field delocalization functions $h(p,y)$, we get:

$$\begin{aligned}
A^\pm_\mu(p,y) &= \tilde{g}h_\pm(p,y)\tilde{W}^\pm_\mu(p)\,,\\
A^3_\mu(p,y) &= \tilde{g}h_W(p,y)\tilde{W}^3_\mu(p) + \tilde{g}'h_Y(p,y)\tilde{\mathcal{Y}}_\mu(p)\\
&= \tilde{e}h_\gamma(p,y)\tilde{A}_\mu(p) + \frac{\tilde{e}}{s_{\tilde{\theta}}c_{\tilde{\theta}}}h_Z(p,y)\tilde{Z}_\mu(p)\,,
\end{aligned} \tag{2.9}$$

with $h_\gamma = h_W + h_Y$ and $h_Z = c^2_{\tilde{\theta}}h_W - s^2_{\tilde{\theta}}h_Y$.

From the boundary conditions (2.5) we get the boundary values for the functions $h(p,y)$:

$$\begin{aligned}
h_\pm(p,y)|_{y=0} &= h_W(p,y)|_{y=0} = 1, & h_Y(p,y)|_{y=0} &= 0\,,\\
h_\pm(p,y)|_{y=\pi R} &= h_W(p,y)|_{y=\pi R} = 0, & h_Y(p,y)|_{y=\pi R} &= 1\,.
\end{aligned} \tag{2.10}$$

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By substituting the solutions (2.9) in the holographic Lagrangian density (2.6), we can compare the result with the generic extension of the quadratic SM gauge Lagrangian written in terms of the two point functions

$$\begin{aligned}\mathcal{L}^{(2)}_{\rm eff} &= \tilde{g}'^2\Pi_{YY}(p^2)\tilde{\mathcal{Y}}_\mu\tilde{\mathcal{Y}}^\mu + \tilde{g}^2\Pi_{WW}(p^2)\tilde{W}^3_\mu\tilde{W}^{3\mu}\\ &+\tilde{g}\tilde{g}'\Pi_{WY}(p^2)\tilde{W}^3_\mu\tilde{\mathcal{Y}}^\mu + 2\tilde{g}^2\Pi_{+-}(p^2)\tilde{W}^+_\mu\tilde{W}^{-\mu}\,. \end{aligned}\tag{2.11}$$

Therefore we get:

$$\begin{aligned}\Pi_{WY}(p^2) &= -\frac{1}{2g_5^2}[h_Y h'_W + h_W h'_Y]_0^{\pi R}, \qquad &\Pi_{YY}(p^2) = -\frac{1}{2g_5^2}[h_Y h'_Y]_0^{\pi R},\\ \Pi_{WW}(p^2) &= -\frac{1}{2g_5^2}[h_W h'_W]_0^{\pi R}, \qquad &\Pi_{+-}(p^2) = -\frac{1}{2g_5^2}[h_+ h'_-]_0^{\pi R}.\end{aligned}\tag{2.12}$$

where $h' = \partial_y h$.

The solutions of the equations of motions (2.2) give $h_\pm(p,y) \equiv h_W(p,y)$ and therefore $\Pi_{WW}(p^2) \equiv \Pi_{+-}(p^2)$, as a consequence of the custodial SU(2) symmetry of the model.

Concerning the oblique corrections, we plug the eqs. (2.12) in the ϵ parameter expressions given in terms of the vacuum polarization amplitudes [33, 31]

$$\epsilon_1^{\rm oblique} = \tilde{g}^2\frac{\Pi_{WW}(0) - \Pi_{+-}(0)}{\tilde{M}_W^2}\,, \tag{2.13}$$

$$\epsilon_2^{\rm oblique} = \tilde{g}^2\frac{d}{dp^2}(\Pi_{+-}(p^2) - \Pi_{WW}(p^2))|_{p^2=0}\,, \tag{2.14}$$

$$\epsilon_3^{\rm oblique} = \tilde{g}^2\frac{d}{dp^2}\Pi_{WY}(p^2)|_{p^2=0}\,, \tag{2.15}$$

obtaining

$$\epsilon_1^{\rm oblique} = \epsilon_2^{\rm oblique} = 0, \qquad \epsilon_3^{\rm oblique} = -\frac{\tilde{g}^2}{2g_5^2}\frac{d}{dp^2}[h_Y h'_W + h_W h'_Y]_{0,p^2=0}^{\pi R}\,. \tag{2.16}$$

This shows how $\epsilon_3^{\rm oblique}$ can be computed by knowing the wave functions of the gauge bosons and their y-derivatives at the extremes of the segment.

Of course it is possible to get the two point functions without the holographic prescription; in this way we would find that the electroweak parameters can be expressed as integrals along the extra dimension. For example:

$$\epsilon_3^{\rm oblique} = \frac{\tilde{g}^2}{g_5^2}\int_0^{\pi R} dy[h_Y h_W]_{p^2=0}\,. \tag{2.17}$$

Using the bulk equations of motion, the boundary conditions and integrating by parts, this integral form for the $\epsilon_3^{\rm oblique}$ parameter turns out to be equivalent to the one expressed as boundary terms in eq. (2.16). Notice that in the boundary formulation of the $\epsilon_3^{\rm oblique}$ we need only the boundary values of the h functions and their derivatives at the first order in p^2 whereas in the integral formulation we need the whole bulk profile of the h functions at zero order in p^2.

In the next section we will consider the fermionic content of the model. Since fermions are delocalized into the bulk, vertex corrections occur and the direct contributions to the electroweak parameters must be taken into account. As a consequence, the estimation of the ϵ parameters is obtained through a general formulation involving the renormalization of the electroweak observables used in the definition of the electroweak parameters as described in [56, 57]. The new physics corrections to the quadratic part of the SM Lagrangian can be encoded in the z coefficients defined as follows:

$$\mathcal{L}^{(2)}_{\text{eff}} = \frac{p^2}{2}(1+z_\gamma)\tilde{A}_\mu\tilde{A}^\mu + p^2(1+z_W)\tilde{W}^+_\mu\tilde{W}^{-\mu} + \frac{p^2}{2}(1+z_Z)\tilde{Z}_\mu\tilde{Z}^\mu - p^2 z_{Z\gamma}\tilde{A}_\mu\tilde{Z}^\mu$$
$$-\tilde{M}^2_W\tilde{W}^+_\mu\tilde{W}^{-\mu} - \frac{1}{2}\tilde{M}^2_Z\tilde{Z}_\mu\tilde{Z}^\mu \,. \qquad (2.18)$$

Then, comparing with eq. (2.11) and performing the standard change of basis in the neutral sector of the gauge fields, we can express the z corrections in terms of the two point functions:

$$z_\gamma = 2\tilde{e}^2\frac{d}{dp^2}\Pi_{\gamma\gamma}(p^2)|_{p^2=0}, \qquad z_W = 2\frac{\tilde{e}^2}{s^2_{\tilde{\theta}}}\frac{d}{dp^2}\Pi_{+-}(p^2)|_{p^2=0},$$
$$z_Z = 2\frac{\tilde{e}^2}{c^2_{\tilde{\theta}}s^2_{\tilde{\theta}}}\frac{d}{dp^2}\Pi_{ZZ}(p^2)|_{p^2=0}, \qquad z_{Z\gamma} = -\frac{\tilde{e}^2}{c_{\tilde{\theta}}s_{\tilde{\theta}}}\frac{d}{dp^2}\Pi_{Z\gamma}(p^2)|_{p^2=0}, \qquad (2.19)$$

whereas the unrenormalized gauge boson masses are given by

$$\tilde{M}^2_W = -2\frac{\tilde{e}^2}{s^2_{\tilde{\theta}}}\Pi_{+-}(0), \quad \tilde{M}^2_Z = -2\frac{\tilde{e}^2}{c^2_{\tilde{\theta}}s^2_{\tilde{\theta}}}\Pi_{ZZ}(0)\,, \qquad (2.20)$$

while the photon is massless $M^2_\gamma = -2\tilde{e}^2\Pi_{\gamma\gamma}(0) \equiv 0$.

All these parameters can be expressed in boundary form thanks to the holographic formulation given in eq. (2.12).

Let us note that, since the unbroken $U_{\text{em}}(1)$ gauge symmetry guarantees that $\Pi_{\gamma\gamma}(0) = 0$, using the solutions of the bulk equations of motion and the boundary conditions for h_W and h_Y, the following relation at $p^2 = 0$ holds:

$$h_\gamma(0,y) = h_W(0,y) + h_Y(0,y) \equiv 1\,. \qquad (2.21)$$

This relation will be used in section 4 for the derivation of the direct contributions to the precision electroweak parameters due to the delocalization of the fermions in the bulk.

2.2 Explicit calculations

Let us now evaluate the electroweak parameters with the explicit solutions of the transverse components of the bulk gauge fields. The integral expression (2.17) for the ϵ_3 parameter makes the analogy with the deconstruction procedure much more direct. In fact in the integral expression (2.17) for the ϵ_3 parameter, the h functions are involved at the zero order in p^2. So we only need to solve the second order differential equations (2.2) at $p^2 = 0$ and to impose the boundary conditions on the branes. The solutions are:

$$h_Y(0,y) = \frac{y}{\pi R}, \qquad h_\pm(0,y) = h_W(0,y) = 1 - \frac{y}{\pi R}\,. \qquad (2.22)$$

These functions are the analogous, in the continuum limit, of the variables y_i and $z_i = 1-y_i$ of the deconstructed formulation of the model given in [28], for which one finds

$$\epsilon_3^{\text{oblique}} = \tilde{g}^2 \sum_{i=1}^{K} \frac{y_i}{g_i^2}(1-y_i)\,, \tag{2.23}$$

where K is the number of sites, g_i are the gauge coupling constants of the replicated gauge groups along the moose chain and $y_i = \sum_{j=1}^{i} f^2/f_j^2$ with f_j the link variables and $1/f^2 = \sum_{j=1}^{K} 1/f_j^2$.

By using in (2.23) the matching between the 5D parameters of the discretized theory (the gauge coupling g_5, the lattice spacing a) and the parameters of the 4D deconstructed theory (the gauge coupling constant along the chain g_j, the link couplings f_j), namely [49]:

$$\frac{a}{g_5^2} \longleftrightarrow \frac{1}{g_j^2}\,, \qquad\qquad \frac{1}{a g_5^2} \longleftrightarrow f_j^2\,, \tag{2.24}$$

and performing the continuum limit, we get the correspondence between the two descriptions in the case of equal gauge couplings along the chain. In such a case the variable y_i is the discretized analogous of the coordinate y of the fifth dimension in the eq. (2.17).

By substituting eq. (2.22) in (2.17), we recover the well known result [26, 28]:

$$\epsilon_3^{\text{oblique}} = \frac{\tilde{g}^2}{g_5^2}\frac{\pi R}{6}\,. \tag{2.25}$$

It is also easy to find out the exact solutions of the bulk equations of motions for the interpolating field delocalization functions. They take the form:

$$h_Y(p,y) = \frac{\sin[py]}{\sin[p\pi R]}, \quad h_\pm(p,y) = h_W(p,y) = \frac{\sin[p(\pi R - y)]}{\sin[p\pi R]}\,, \tag{2.26}$$

from which it is straightforward to get the same result given in eq. (2.25) for the electroweak parameter $\epsilon_3^{\text{oblique}}$, by using the boundary expression (2.16).

Moreover, by using the exact solutions (2.26) we can compute the two point functions defined in eq. (2.12), and, by comparing with eqs. (2.19)–(2.20), we can extract the unrenormalized masses:

$$\tilde{M}_Z = \frac{v}{2}\frac{\tilde{g}}{c_{\tilde{\theta}}}, \quad \tilde{M}_W = \frac{v}{2}\tilde{g}\,, \tag{2.27}$$

where $v \equiv \frac{2}{g_5\sqrt{\pi R}}$, and the z corrections:

$$\begin{aligned} z_\gamma &= \frac{\tilde{g}^2 \pi R s_{\tilde{\theta}}^2}{g_5^2}, & z_W &= \frac{\tilde{g}^2 \pi R}{3 g_5^2}, \\ z_Z &= \frac{\tilde{g}^2 \pi R (c_{\tilde{\theta}}^4 - c_{\tilde{\theta}}^2 s_{\tilde{\theta}}^2 + s_{\tilde{\theta}}^4)}{3 c_{\tilde{\theta}}^2 g_5^2}, & z_{Z\gamma} &= \frac{\tilde{g}^2 \pi R s_{\tilde{\theta}}(-c_{\tilde{\theta}}^2 + s_{\tilde{\theta}}^2)}{2 c_{\tilde{\theta}} g_5^2}\,. \end{aligned} \tag{2.28}$$

The above expressions are in agreement with those obtained by performing the continuum limit of the deconstructed moose model in [36].

Let us notice that the additional electroweak parameters introduced in [17], namely X, Y, W, are suppressed by a factor $M_W^2 R^2$ with respect to S, while $V = 0$ due to the custodial symmetry of the model.

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3. Holographic analysis of the fermionic sector

For what concerns fermions in one extra dimension we can carry out the procedure given in [50], starting from the following bulk action for the 5D Dirac field, in the unitary gauge ($A_5 = 0$):

$$S^{\rm bulk}_{\rm ferm} = \frac{1}{\hat{g}_5^2}\int d^4x \int_0^{\pi R} dy \left\{ i\bar{\Psi}\gamma^\mu D_\mu \Psi + \frac{1}{2}[\bar{\Psi}\gamma^5\partial_5\Psi - \partial_5\bar{\Psi}\gamma^5\Psi] - M\bar{\Psi}\Psi \right\}, \tag{3.1}$$

where $\hat{g}_5$ is a parameter - in general different from the bulk gauge coupling g_5 - with mass dimension $-1/2$, M is a constant mass for the bulk fermions and

$$D_\mu \Psi(x,y) = \Big(\partial_\mu + iT_a A^a_\mu(x,y) + iY_L A^3_\mu(x,\pi R)\Big)\Psi(x,y)\,, \tag{3.2}$$

with $Y_L = (B-L)/2$ the left hypercharge. The hypercharge contribution to the covariant derivative appears as a non-local term along the extra dimension since it is evaluated in $y = \pi R$.

Performing the variational analysis of the fermionic action, the bulk equations of motion for a free Dirac field written in terms of the left and right handed components: $\Psi = \psi_L + \psi_R$ with $\gamma_5\psi_{L,R} = \mp\psi_{L,R}$, are, in the momentum space:

$$\not{p}\psi_L(p,y) + (\partial_5 - M)\psi_R(p,y) = 0, \qquad \not{p}\psi_R(p,y) - (\partial_5 + M)\psi_L(p,y) = 0\,. \tag{3.3}$$

The left and right handed components of the bulk Dirac field result to be described by a system of two coupled first order differential equations that mix the two chiral components: however the system can be decoupled acting on the previous eqs. (3.3) with the operators $(\partial_5 + M)$ and $(\partial_5 - M)$ respectively. Then, both right and left handed fields satisfy the second order differential equation

$$(\partial_5^2 + \omega^2)\psi_{L,R} = 0 \quad \text{where} \quad \omega = \sqrt{p^2 - M^2}\,. \tag{3.4}$$

3.1 Boundary conditions for fermions

We generalize the procedure described in the gauge sector to determine the boundary values for bulk fermions; in fact, following [58], we add to the bulk action (3.1) the brane action

$$\begin{aligned} S^{\rm brane}_{\rm ferm} = \int d^4x \int_0^{\pi R} dy \delta(y) & \left[\bar{q}_L i\gamma^\mu D_\mu q_L + \frac{1}{\hat{g}_5^2}\left(\mathtt{t}_L(\bar{\psi}_R q_L + \bar{q}_L\psi_R) - \frac{1}{2}\bar{\Psi}\Psi\right)\right] \\ & + \delta(y-\pi R)\left[\bar{q}_R i\gamma^\mu D_\mu q_R + \frac{1}{\hat{g}_5^2}\left(\mathtt{t}_R(\bar{q}_R\psi_L + \bar{\psi}_L q_R) - \frac{1}{2}\bar{\Psi}\Psi\right)\right], \end{aligned} \tag{3.5}$$

which contains kinetic terms for the interpolating brane fields q_L and q_R, their couplings to the bulk Dirac field Ψ and pseudo-mass terms for the bulk fermion field. In agreement with the gauge symmetries on the branes, we have

$$\begin{aligned} D_\mu q_L|_{y=0} &= \Big(\partial_\mu + i\tilde{g}T^a\tilde{W}^a_\mu + i\tilde{g}'Y_L\tilde{\mathcal{Y}}_\mu\Big)\, q_L\,, \\ D_\mu q_R|_{y=\pi R} &= \Big(\partial_\mu + i\tilde{g}'T^3\tilde{\mathcal{Y}}_\mu + i\tilde{g}'Y_L\tilde{\mathcal{Y}}_\mu\Big)\, q_R\,. \end{aligned} \tag{3.6}$$

We have allowed different couplings $\mathtt{t}_{L,R}$ between the right and left handed brane fields and the bulk fermions, [51, 49]. They parameterize the delocalization in the bulk of the brane fermions and, as we will see, are responsible for the fermion masses. The couplings $\mathtt{t}_{L,R}$ can in general be different for each flavor in order to reproduce the fermion mass spectrum. Since this is not the aim of this paper, for sake of simplicity, we will assume universal $\mathtt{t}_{L,R}$ and an implicit sum over the flavors.

Let us now perform the variational analysis on the branes for the total action $S^{\rm bulk}_{\rm ferm}+S^{\rm brane}_{\rm ferm}$ with fixed fields $\delta q_L=\delta q_R\equiv 0$. The coefficients of the variations $\delta\bar\psi_R$ in $y=\pi R$ and of $\delta\bar\psi_L$ in $y=0$ are automatically vanishing whereas the coefficients of the variation $\delta\bar\psi_L$ in $y=\pi R$ and $\delta\bar\psi_R$ in $y=0$ fix the boundary values of the two chiral bulk spinors:

$$\psi_L(p,0)\equiv \mathtt{t}_L q_L(p), \qquad \psi_R(p,\pi R)\equiv \mathtt{t}_R q_R(p)\,. \tag{3.7}$$

Thus, the degrees of freedom in terms of which we can eliminate the bulk field in the holographic prescription are the 4D fields q_L and q_R which, indeed, live on the branes $y=0$ and $y=\pi R$ respectively. Note that, with this choice, we get the same scenario of the moose model with direct couplings for the fermions [36], where q_L and q_R correspond to the standard left and right handed fermions.

Once we have fixed the boundary values of the bulk fields we can determine the explicit solutions of the differential equations (3.3) with boundary conditions (3.7), which are

$$\begin{aligned}\psi_L(p,y) &= f_L(p,y)\mathtt{t}_L q_L(p)+\not p\pi R\tilde f_L(p,y)\mathtt{t}_R q_R(p),\\ \psi_R(p,y) &= f_R(p,y)\mathtt{t}_R q_R(p)+\not p\pi R\tilde f_R(p,y)\mathtt{t}_L q_L(p)\,,\end{aligned} \tag{3.8}$$

with

$$\begin{aligned}f_L(p,y) &= \frac{\omega\cos[\omega(\pi R-y)]+M\sin[\omega(\pi R-y)]}{\omega\cos[\pi R\omega]+M\sin[\pi R\omega]},\\ \tilde f_L(p,y) &= \frac{1}{\pi R}\frac{\sin[\omega y]}{\omega\cos[\pi R\omega]+M\sin[\pi R\omega]},\\ f_R(p,y) &= \frac{\omega\cos[\omega y]+M\sin[\omega y]}{\omega\cos[\pi R\omega]+M\sin[\pi R\omega]},\\ \tilde f_R(p,y) &= \frac{1}{\pi R}\frac{\sin[\omega(\pi R-y)]}{\omega\cos[\pi R\omega]+M\sin[\pi R\omega]}\,.\end{aligned} \tag{3.9}$$

At $O(p^2)$ we get:

$$\begin{aligned}\psi_L(p,y) &\sim \mathtt{t}_L q_L(p)e^{-My}+\not p\mathtt{t}_R q_R(p)\frac{\sinh[My]}{M}e^{-M\pi R},\\ \psi_R(p,y) &\sim \mathtt{t}_R q_R(p)e^{M(y-\pi R)}+\not p\mathtt{t}_L q_L(p)\frac{\sinh[M(y-\pi R)]}{M}e^{-M\pi R}\,.\end{aligned} \tag{3.10}$$

Let us note that we have non vanishing right and left handed contributions in the $y=0$ and $y=\pi R$ branes respectively. The particular solution with null bulk mass, $M=0$, is

$$\psi_L(p,y)\sim \mathtt{t}_L q_L(p)+\not p y\mathtt{t}_R q_R(p), \quad \psi_R(p,y)\sim \mathtt{t}_R q_R(p)+\not p\mathtt{t}_L q_L(p)(\pi R-y). \tag{3.11}$$

Hence the two components of the Dirac field have a flat profile along the extra dimension at zero order in p. A study of the phenomenological implications of the M parameter is developed in [50].

The effective Lagrangian can be deduced after the normalization of the kinetic term as we will see in the following section.

4. The interaction

So far we have derived the holographic description of the fermionic sector by imposing the chiral equations of motion (3.3) and the boundary conditions (3.7), but we have not considered the interaction with the gauge fields. In fact, in presence of covariant derivatives in the bulk action (3.1), the interaction terms are not eliminated by the equations of motion. By applying the holographic prescription, the residual fermionic action terms of the theory are

$$S_{\rm ferm}^{holog+brane} = S_{\rm ferm}^{\rm brane} - \int \frac{d^4p}{(2\pi)^4} \int_0^{\pi R} \frac{dy}{\hat{g}_5^2} \bar{\Psi}(p,y)\gamma^\mu \left[A_\mu(p,y) + \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu(p) \right] \Psi(p,y)\,, \tag{4.1}$$

where $S_{\rm ferm}^{\rm brane}$ is evaluated by using the (3.8) solutions.

Let us observe that the $B-L$ interaction described in eq. (4.1), coming from the covariant derivative term (3.2), appears as a non local interaction term in the fifth dimension. A way to generate this interaction through a local bulk dynamics is to introduce an additional gauge symmetry $\mathrm{U}(1)_{B-L}$ in the bulk with gauge coupling g'_5. In analogy with eq. (2.4), the related bulk field $B(x,y)$ is fixed on both the boundaries by brane mass terms (in the limit of large mass) in order to obtain the boundary conditions $B(x,y)|_{y=0,\pi R} = \tilde{g}'\tilde{\mathcal{Y}}$. Since the boundary value for the $B(x,y)$ is equal on both branes, its delocalization function $h_{B-L}(p,y)$ at $p^2=0$ is flat, that is $h_{B-L}(0,y)=1$. Therefore, the Dirac bulk field has a local 5D interaction that, at effective level, reproduces the one given in eq. (4.1). In fact, as we shall see in the following analysis, the low energy effective interaction Lagrangian at leading order in p^2 is described in terms of the delocalization functions $h(0,y)$. Moreover, the holographic term of the $\mathrm{U}(1)_{B-L}$ bulk theory, analogous of the one in eq. (2.3),

$$S_{B-L}^{\rm holog} = -\frac{1}{2g'^2_5} \int \frac{d^4p}{(2\pi)^4} [B_\mu(p,y)\partial_5 B^\mu(p,y)]_0^{\pi R} \tag{4.2}$$

can be neglected if we suppose $g'_5 \gg g_5$. In conclusion, the low energy limit of the model with a local bulk $\mathrm{U}(1)_{B-L}$ interaction, is the same of the non local one that we have studied. However the model with the additional $\mathrm{U}(1)_{B-L}$ symmetry in the bulk and its phenomenological implications deserve a dedicated study, even though the oblique corrections to the ϵ_3 parameter are unaffected by the extra $B-L$ factor [48].

In order to evaluate the low energy effective Lagrangian, we plug the generic solution of the equations of motion, eqs. (3.8), into the eq. (4.1). Neglecting again the $p \cdot A$ terms since we are considering only the transverse components of the bulk field, we get

$$\begin{aligned}
\mathcal{L}_{\text{ferm}}^{holog+brane} &= \frac{\pi R}{\hat{g}_5^2}\mathtt{t}_L^2 \tilde{f}_R(p,0)\bar{q}_L(p)\ \not{p} q_L(p) + \frac{\pi R}{\hat{g}_5^2}\mathtt{t}_R^2 \tilde{f}_L(p,\pi R)\bar{q}_R(p)\ \not{p} q_R(p) \\
&+ \frac{\mathtt{t}_L \mathtt{t}_R}{2\hat{g}_5^2}\left[f_R(p,0) + f_L(p,\pi R)\right]\left(\bar{q}_L(p)q_R(p) + \bar{q}_R(p)q_L(p)\right) \\
&+ \bar{q}_L(p)\gamma^\mu \left[p_\mu - \tilde{g}\tilde{W}_\mu(p) - \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu(p)\right] q_L(p) \\
&+ \bar{q}_R(p)\gamma^\mu \left[p_\mu - \tilde{g}'\tilde{Y}_\mu(p) - \frac{\tilde{g}'(B-L)}{2}\tilde{\mathcal{Y}}_\mu(p)\right] q_R(p) \\
&- \int_0^{\pi R}\frac{dy}{\hat{g}_5^2}\left(\mathtt{t}_L^2 f_L^2(p,y) + (p\pi R)^2 \mathtt{t}_L^2 \tilde{f}_R^2(p,y)\right)\bar{q}_L(p)\gamma^\mu \times \\
&\qquad\qquad \times \left[A_\mu(p,y) + \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu(p)\right] q_L(p) \\
&- \int_0^{\pi R}\frac{dy}{\hat{g}_5^2}\left(\mathtt{t}_R^2 f_R^2(p,y) + (p\pi R)^2 \mathtt{t}_R^2 \tilde{f}_L^2(p,y)\right)\bar{q}_R(p)\gamma^\mu \times \\
&\qquad\qquad \times \left[A_\mu(p,y) + \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu(p)\right] q_R(p)\,.
\end{aligned}$$

where $\tilde{Y}_\mu = \tilde{\mathcal{Y}}_\mu T^3$. Then we will use the generic solutions of the bulk equations of motion for the bulk gauge fields given in eqs. (2.9), in order to deal only with the gauge fields $\tilde{W}$ and $\tilde{\mathcal{Y}}$. Notice that, in presence of direct couplings of the bulk gauge fields to standard fermions, since the simplest holographic approach consists in solving the free equations of motion for the fields, effective fermion current-current interactions, which are obtained in the deconstruction analysis [36], are not recovered. The full effective action could be built solving the complete bulk equations of motion with a suitable perturbative expansion [39]. As pointed out in the numerical analysis performed in [36], the current-current terms, in the region of the parameter space allowed by the precision electroweak data, are negligible. For this reason we have only considered the free equations of motion for the bulk fields.

Notice also that, within this approach, the kinetic terms for the fermions no longer come from the bulk action but they come from the boundary kinetic terms and from the pseudo-mass terms of the brane action $S_{\text{ferm}}^{\text{brane}}$, that is from the terms of the type $\bar{q}\Psi$ or $\bar{\Psi}\Psi$ in eq. (3.5). This implies that we have, at $O(p^2)$,

$$\mathcal{L}_{\text{ferm}}^{\text{kin}} \sim \bar{q}_L\ \not{p}\left(1 + \mathtt{t}_L^2\frac{\pi R}{\hat{g}_5^2}\tilde{f}_R(0,0)\right) q_L + \bar{q}_R\ \not{p}\left(1 + \mathtt{t}_R^2\frac{\pi R}{\hat{g}_5^2}\tilde{f}_L(0,\pi R)\right) q_R\,. \tag{4.3}$$

In order to have canonical kinetic terms, a normalization of the brane interpolating fields is necessary:

$$q_L \to \frac{q_L}{\sqrt{1 + \mathtt{t}_L^2\frac{\pi R}{\hat{g}_5^2}\tilde{f}_R(0,0)}}\,, \qquad q_R \to \frac{q_R}{\sqrt{1 + \mathtt{t}_R^2\frac{\pi R}{\hat{g}_5^2}\tilde{f}_L(0,\pi R)}}\,. \tag{4.4}$$

Using the properties of the f and $\tilde{f}$ functions given in appendix A, the normalization factor can be written in integral form

$$q_{L,R} \to \frac{q_{L,R}}{\sqrt{1 + \int_0^{\pi R} dy\ b_{L,R}(y)}}\,, \tag{4.5}$$

where

$$b_{L,R}(y) = \mathtt{t}^2_{L,R}\frac{f^2_{L,R}(0,y)}{\hat{g}_5^2}\,. \tag{4.6}$$

In the form (4.5) the relation between the holographic procedure and the continuum limit of the deconstructed version of the model is much more evident since $\int dy \to a\sum_{i=1}^K$, $b_L(y) \to b_i/a$ and $b_R(y) \to b'_i/a$ where a is the lattice spacing [36, 49].

After the normalization (4.4), defining the electric charge as $Q = T^3 + \frac{B-L}{2}$, and using eq. (2.21), we can extract from the effective Lagrangian (4.3) the lowest order interaction terms:

$$\begin{aligned}\mathcal{L}_{\text{ferm}} = &-\tilde{e}Q\tilde{A}_\mu\bar{q}\gamma^\mu q - \frac{\tilde{e}}{s_{\tilde{\theta}}c_{\tilde{\theta}}}\tilde{Z}_\mu\bar{q}\gamma^\mu\left\{T^3\frac{1-\gamma_5}{2}\left(1-\frac{\mathtt{b}_L}{2}\right) - T^3\frac{1+\gamma_5}{2}\frac{\mathtt{b}_R}{2} - s^2_{\tilde{\theta}}Q\right\}q\\ &-\left[\frac{\tilde{e}}{s_{\tilde{\theta}}\sqrt{2}}\tilde{W}^-_\mu\bar{q}_d\gamma^\mu\left\{\frac{1-\gamma_5}{2}\left(1-\frac{\mathtt{b}_L}{2}\right) - \frac{1+\gamma_5}{2}\frac{\mathtt{b}_R}{2}\right\}q_u + h.c.\right],\end{aligned} \tag{4.7}$$

where $q = q_L + q_R$, and the corrections to the electroweak currents are given by

$$\mathtt{b}_L = \frac{2\int_0^{\pi R} dy\; b_L(y)h_Y(0,y)}{1+\int_0^{\pi R} dy\; b_L(y)},\quad \mathtt{b}_R = \frac{2\int_0^{\pi R} dy\; b_R(y)h_Y(0,y)}{1+\int_0^{\pi R} dy\; b_R(y)}\,. \tag{4.8}$$

The $\mathtt{b}_R$ parameter gives rise to charged and neutral right handed currents coupled with the SM gauge bosons and for this reason it has phenomenologically strong bounds related to the $b \to s\gamma$ process, [59], and the μ decay, [60]. Allowing different brane coupling coefficients $\mathtt{t}_L$ and $\mathtt{t}_R$ for the q_L and q_R four dimensional fermions, we get different values for $\mathtt{b}_L$ and $\mathtt{b}_R$. In particular, a small brane coupling coefficient $\mathtt{t}_R$ with respect to $\mathtt{t}_L$ suppress the $\mathtt{b}_R$ contribution. In the following phenomenological analysis we will neglect the $\mathtt{t}_R$ contribution.

After identifying the physical parameters as in [57] and following the procedure used in [36], that is identifying the physical fields by diagonalizing $\mathcal{L}^{(2)}_{\text{eff}}$ in eq. (2.18),

$$\tilde{A}_\mu = \left(1-\frac{z_\gamma}{2}\right)A_\mu + z_{Z\gamma}Z_\mu\,,\ \tilde{W}^\pm_\mu = \left(1-\frac{z_W}{2}\right)W^\pm_\mu\,,\ \tilde{Z}_\mu = \left(1-\frac{z_Z}{2}\right)Z_\mu\,, \tag{4.9}$$

we can derive the expression for the ϵ_3 parameter, including the oblique and direct contributions. By taking only the leading order in $\mathtt{t}^2_L\pi R/\hat{g}_5^2$ and in the limit $\tilde{g}^2\pi R/g_5^2 \ll 1$, corresponding to $\tilde{g}^2/g_i^2 \ll 1$ in the deconstructed version, we get:

$$\epsilon_3 = \int_0^{\pi R} dy h_Y(0,y)\left\{\frac{\tilde{g}^2}{g_5^2}h_W(0,y) - b_L(y)\right\}\,. \tag{4.10}$$

The fermion contribution contains $b_L(y)$, given in eq. (4.6), and turns out to be proportional to the square of the left-handed fermion interpolating function at leading order in p^2. Notice that, as already stated for the oblique corrections, also the direct contribution to ϵ_1 and ϵ_2 parameters vanishes because the corrections to the fermionic currents do not violate the custodial SU(2) symmetry of the model.

Eq. (4.10) is the continuum analogous of the ϵ_3 parameter found in the linear moose with direct couplings of the left handed fermions, [36].

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The ideal fermionic delocalization, corresponding to vanishing ϵ_3, that is the bulk profile $f_L(0,y)$ that makes the integrand of eq. (4.10) null in every bulk point y, is related to the bulk delocalization profile h_W of the $\tilde{W}$ interpolating field through the following relation [1]

$$b_L(y) = \mathtt{t}_L^2 \frac{f_L^2(0,y)}{\hat{g}_5^2} = \frac{\tilde{g}^2}{g_5^2} h_W(0,y) \quad \forall y \in [0, \pi R]\,. \tag{4.11}$$

Using the explicit solution for h_W given in eq. (2.22), we find that the ideal delocalization should be given by

$$f_L(0,y) \propto \sqrt{1 - \frac{y}{\pi R}}\,. \tag{4.12}$$

However this ideal delocalization is not allowed by the equations of motion for the Dirac bulk field independently from the assumed bulk mass as can be checked by eq. (3.9). This result has already been found in [49].

Since the ideal delocalization is not allowed, the remaining possibility to get a zero new physics contribution to ϵ_3 is to ask for a global cancellation, that is a vanishing ϵ_3 without requiring the integrand of eq. (4.10) to be zero. This links the parameters of the gauge sector to the fermionic ones as shown in [34, 35, 49].

As a last point we note that, by keeping $\mathtt{t}_R \neq 0$, after the normalization (4.5) of the interpolating fermionic fields, we get the following Dirac mass term

$$\mathcal{L}_{\text{ferm}}^{\text{mass}} = \frac{1}{2} \frac{\mathtt{t}_R \mathtt{t}_L}{\hat{g}_5^2} \left[\frac{f_R(0,0) + f_L(0,\pi R)}{\sqrt{1 + \int_0^{\pi R} dy\ b_L(y)}\sqrt{1 + \int_0^{\pi R} dy\ b_R(y)}} \right] (\bar{q}_L q_R + \bar{q}_R q_L)\,. \tag{4.13}$$

Since we are working in the limit $\mathtt{t}_{L,R}^2 \pi R/\hat{g}_5^2 \ll 1$ we find that the 4D mass is $m = \frac{\mathtt{t}_L \mathtt{t}_R}{\hat{g}_5^2} \exp(-M\pi R)$. As already noticed, [51, 49], assuming the ϵ_3 global cancellation, the top mass value cannot be reproduced without allowing for a microscopical Lorentz invariance breaking along the extra dimension.

5. Warped scenario

5.1 Holographic analysis for the gauge sector

Let us now extend the holographic analysis for the gauge sector to the case of Randall-Sundrum (RS) metric with warp factor k:

$$ds^2 = \frac{1}{(kz)^2}(dx^2 - dz^2)\,, \tag{5.1}$$

so that

$$S_{\text{YM}}^{\text{bulk}} = -\frac{1}{2g_5^2} \int d^4x \int_{L_0}^{L_1} dz \left(\frac{1}{kz}\right) \mathsf{Tr}[F^{MN}(x,z) F_{MN}(x,z)]\,, \tag{5.2}$$

where L_0 and L_1 are the brane locations.

[1] A similar relation between the wave function of the ordinary fermions and the wave function of the standard W boson is suggested in [37]

The bulk equations of motion in the momentum space, separating the longitudinal and the transverse components of the gauge field, are

$$(D_5^2 + p^2) A^t_\mu(p,z) = 0, \qquad D_5^2 A^l_\mu(p,z) = 0\,, \tag{5.3}$$

where

$$D_5^2 = z\partial_5 \left(\frac{1}{z} \partial_5 \right). \tag{5.4}$$

Considering only the transverse components and imposing the equations of motion as a constraint to the 5D Yang-Mills action in the RS metric, we are left with the holographic Lagrangian

$$\mathcal{L}^{(2)holog}_{\rm YM} = -\frac{1}{2g_5^2}\frac{1}{kz} A^{a\mu}\partial_5 A^a_\mu|_{z=L_1} + \frac{1}{2g_5^2}\frac{1}{kz} A^{a\mu}\partial_5 A^a_\mu|_{z=L_0}\,, \tag{5.5}$$

where, as in the flat case, we are neglecting the trilinear and quadrilinear terms of this non-abelian 5D SU(2) Yang-Mills theory.

For what concerns the boundary conditions we add to the warped action in eq. (5.2) the same brane terms of the flat case, given in eq. (2.4), so that in the limit $c_{1,2} \to \infty$ we get the same boundary values (2.5) for the bulk field up to a redefinition of the brane locations

$$\begin{aligned} A^\pm_\mu(x,z)|_{z=L_0} &\equiv \tilde{g}\tilde{W}^\pm_\mu(x), \qquad & A^\pm_\mu(x,z)|_{z=L_1} &\equiv 0, \\ A^3_\mu(x,z)|_{z=L_0} &\equiv \tilde{g}\tilde{W}^3_\mu(x), \qquad & A^3_\mu(x,z)|_{z=L_1} &\equiv \tilde{g}'\tilde{\mathcal{Y}}_\mu(x)\,, \end{aligned} \tag{5.6}$$

and a new definition of the gauge couplings in order to keep track of the metric induced on the branes: $\tilde{g} \to \tilde{g}\sqrt{kL_0}$, $\tilde{g}' \to \tilde{g}'\sqrt{kL_1}$.

Imposing the bulk equations of motion (5.3), and the boundary values of the bulk field on the branes (5.6), the holographic formulation of the model in warped space-time, including also the brane kinetic terms, is

$$\begin{aligned} \mathcal{L}^{(2)holog+SM}_{\rm YM} = &-\frac{\tilde{g}'}{2g_5^2}\left[\frac{1}{kz}\tilde{\mathcal{Y}}^\mu(p)\partial_5 A^3_\mu(p,z)\right]_{z=L_1} + \frac{\tilde{g}}{2g_5^2}\left[\frac{1}{kz}\tilde{W}^{a\mu}(p)\partial_5 A^a_\mu(p,z)\right]_{z=L_0} \\ &+\frac{p^2}{2}\tilde{W}^a_\mu(p)\tilde{W}^{a\mu}(p) + \frac{p^2}{2}\tilde{\mathcal{Y}}_\mu(p)\tilde{\mathcal{Y}}^\mu(p)\,. \end{aligned} \tag{5.7}$$

By expressing the generic solutions of the bulk equations of motion in terms of the delocalization functions as in eq. (2.9), the warped analogous of the vacuum polarization functions, given in eqs. (2.12), contain the warp factor $\frac{1}{kz}$, that is:

$$\begin{aligned} \Pi_{WY}(p^2) &= -\frac{1}{2g_5^2}\left[\frac{1}{kz}(h_Y h'_W + h_W h'_Y)\right]^{L_1}_{L_0}, \quad & \Pi_{YY}(p^2) &= -\frac{1}{2g_5^2}\left[\frac{1}{kz}(h_Y h'_Y)\right]^{L_1}_{L_0}, \\ \Pi_{WW}(p^2) &= -\frac{1}{2g_5^2}\left[\frac{1}{kz}(h_W h'_W)\right]^{L_1}_{L_0}, & \Pi_{+-}(p^2) &= -\frac{1}{2g_5^2}\left[\frac{1}{kz}(h_+ h'_-)\right]^{L_1}_{L_0}, \end{aligned} \tag{5.8}$$

and the identity $\Pi(p^2)_{WW} \equiv \Pi(p^2)_{\pm\mp}$ still holds due to the custodial symmetry. Hence, the oblique contributions to the ϵ parameters are

$$\epsilon_1^{\rm oblique} = 0, \qquad \epsilon_2^{\rm oblique} = 0, \qquad \epsilon_3^{\rm oblique} = -\frac{\tilde{g}^2}{2g_5^2}\frac{d}{dp^2}[\frac{1}{kz}(h_Y h'_W + h_W h'_Y)]^{L_1}_{L_0, p^2=0}\,. \tag{5.9}$$

Also in the warped scenario the $\epsilon_3^{\rm oblique}$ parameter can be expressed in integral form, that is

$$\epsilon_3^{\rm oblique} = \frac{\tilde{g}^2}{g_5^2}\int_{L_0}^{L_1} dz \frac{1}{kz}[h_Y h_W]_{p^2=0}\,, \tag{5.10}$$

for which it is sufficient to solve the equations of motion at $p^2 = 0$. In this limit the differential equation to solve is simply $(\frac{1}{kz}h'_{Y(W)})' \equiv 0$, and, imposing the boundary conditions, the solutions are

$$h_Y(0,z) = \frac{L_0^2 - z^2}{L_0^2 - L_1^2}, \qquad h_\pm(0,z) = h_W(0,z) = \frac{L_1^2 - z^2}{L_1^2 - L_0^2}\,. \tag{5.11}$$

In order to evaluate the electroweak parameters coming from the bulk gauge sector we need the exact solutions of the equations of motion (5.3) for the h functions. These are given in terms of the Bessel functions J_1 and Y_1:

$$\begin{aligned} h_Y(p,z) &= \frac{z}{L_1}\frac{J_1[pz]Y_1[pL_0] - J_1[pL_0]Y_1(pz)}{J_1[pL_1]Y_1[pL_0] - J_1[pL_0]Y_1[pL_1]}, \\ h_\pm(p,z) = h_W(p,z) &= \frac{z}{L_0}\frac{J_1[pz]Y_1[pL_1] - J_1[pL_1]Y_1[pz]}{J_1[pL_0]Y_1[pL_1] - J_1[pL_1]Y_1[pL_0]}\,. \end{aligned} \tag{5.12}$$

We can now evaluate the two point functions given in eqs. (5.8). For instance:

$$\Pi_{WY}(p^2) = \frac{2}{kg_5^2\pi L_0 L_1}\frac{1}{J_1[pL_0]Y_1[pL_1] - J_1[pL_1]Y_1[pL_0]}\,, \tag{5.13}$$

and from eq. (5.9), we get

$$\epsilon_3^{\rm oblique} = \frac{\tilde{g}^2}{4kg_5^2}\frac{L_1^4 - L_0^4 - 4L_0^2L_1^2\log[L_1/L_0]}{(L_1^2 - L_0^2)^2}\,. \tag{5.14}$$

Of course this result, which is in agreement with [28], is the same obtained by using the zero order solution (5.11) for the h functions in the integral form (5.10) of the ϵ_3 parameter.

Moreover, we can evaluate the z parameters, which are needed for the determination of the non oblique contributions to ϵ_3 coming from the bulk Dirac sector. Plugging the vacuum amplitudes of the warped case in eqs. (2.19), we obtain

$$\begin{aligned} z_\gamma &= \frac{\tilde{g}^2 s_{\tilde\theta}^2}{kg_5^2}\log\left[\frac{L_1}{L_0}\right] \\ z_W &= -\frac{\tilde{g}^2}{4kg_5^2}\frac{L_0^4 - 4L_0^2L_1^2 + 3L_1^4 + 4L_1^4\log[L_0/L_1]}{(L_0^2 - L_1^2)^2}, \\ z_Z &= \frac{\tilde{g}^2}{4kc_{\tilde\theta}^2 g_5^2}\frac{\left(L_0^4 - L_1^4 - 2(L_0^2 - L_1^2)^2 c_{2\tilde\theta} - 4\log[L_0/L_1]\right)(L_1^2c_{\tilde\theta}^2 + L_0^2 s_{\tilde\theta}^2)^2}{(L_0^2 - L_1^2)^2}, \\ z_{Z\gamma} &= \frac{\tilde{g}^2 s_{\tilde\theta}}{2kc_{\tilde\theta}g_5^2}\frac{L_0^2 - L_1^2 - 2\log[L_0/L_1](L_1^2c_{\tilde\theta}^2 + L_0^2s_{\tilde\theta}^2)}{L_0^2 - L_1^2}\,, \end{aligned} \tag{5.15}$$

and the unrenormalized square masses have the same expression given in (2.20) with $v^2 \equiv 8/(k(L_1^2 - L_0^2)g_5^2)$.

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Using the leading order behavior in p^2 for the functions $h(p, y)$, it can be noted that, both in the flat and in the warped scenario, the eqs. (2.9) reproduce the same solutions obtained with the heavy mode elimination from the equations of motion used in the deconstructed version of the model, [36], extrapolated to the continuum.

5.2 Fermions in warped scenario

Let us now consider fermions in the warped metric in order to find the holographic description and obtain the effective Lagrangian. Defining $c = M/k$, the variation of the bulk action, (for a review see for example [61]), leads to the following bulk equations of motion for the left-handed and the right-handed components of the Dirac field

$$\not{p}\psi_L(p,z) + \left(\partial_5 - \frac{c+2}{z}\right)\psi_R(p,z) = 0, \quad \not{p}\psi_R(p,z) - \left(\partial_5 + \frac{c-2}{z}\right)\psi_L(p,z) = 0\,. \quad (5.16)$$

As in the flat scenario, these first order differential equations can be decoupled in two second order differential equations, one for the left handed spinor and one for the right handed spinor. The solutions are given in terms of Bessel functions as in the warped gauge sector. The boundary conditions are fixed by adding the brane action

$$\begin{aligned} S^{\rm brane}_{\rm ferm} = & \int d^4x \int_{L_0}^{L_1} dz \left\{ \delta(z-L_0)\left[\bar{q}_L i\gamma_\mu D^\mu q_L + \frac{1}{(kz)^4}\frac{1}{\hat{g}_5^2}\left(\mathtt{t}_L(\bar{\psi}_R q_L + \bar{q}_L\psi_R) - \frac{1}{2}\bar{\Psi}\Psi\right)\right]\right. \\ & \left. + \delta(z-L_1)\left[\bar{q}_R i\gamma_\mu D^\mu q_R + \frac{1}{(kz)^4}\frac{1}{\hat{g}_5^2}\left(\mathtt{t}_R(\bar{q}_R\psi_L + \bar{\psi}_L q_R) - \frac{1}{2}\bar{\Psi}\Psi\right)\right]\right\}\,. \end{aligned} \quad (5.17)$$

The values of the bulk fields on the branes are given in terms of the interpolating brane fields

$$\psi_L(p, L_0) \equiv \mathtt{t}_L q_L(p), \qquad \psi_R(p, L_1) \equiv \mathtt{t}_R q_R(p)\,. \quad (5.18)$$

The generic solutions for the bulk Dirac field can always be written in the same form as for the flat case, given in eqs. (3.8), where the dimensional parameter πR, used in order to give the same dimensionality to the f and the $\tilde{f}$, can be thought of, for example, the characteristic length of the extra dimension in the RS metric ($\pi R = L_1 - L_0$).

In the $p \to 0$ limit the two first order differential equations (5.16) are decoupled for the left and right-handed spinors and the corresponding delocalization functions are given by

$$f_L(0, z) = (\frac{z}{L_0})^{2-c}, \qquad f_R(0, z) = (\frac{z}{L_1})^{2+c}\,. \quad (5.19)$$

Following the same procedure as in the flat scenario from the interaction terms, we get:

$$\begin{aligned} S^{holog+brane}_{\rm ferm} = S^{\rm brane}_{\rm ferm} - \int \frac{d^4p}{(2\pi)^4}\int_{L_0}^{L_1}\frac{dz}{\hat{g}_5^2(kz)^4}\bar{\Psi}(p,z)\gamma^\mu \times \\ \times \left[A_\mu(p,z) + \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu(p)\right]\Psi(p,z)\,. \end{aligned} \quad (5.20)$$

It is easy to prove that, at order $O(p^2)$ and with canonically normalized kinetic terms, the neutral and charged interactions are described by the eq. (4.7) where the corrections are given by the same expressions (4.8) as in the flat case, with

$$
\begin{aligned}
b_L(z) &= \mathtt{t}_L^2 \frac{f_L^2(0,z)}{\hat{g}_5^2 (kz)^4} = \frac{\mathtt{t}_L^2}{\hat{g}_5^2} \left(\frac{1}{kL_0}\right)^4 \left(\frac{L_0}{z}\right)^{2c}, \\
b_R(z) &= \mathtt{t}_R^2 \frac{f_R^2(0,z)}{\hat{g}_5^2 (kz)^4} = \frac{\mathtt{t}_R^2}{\hat{g}_5^2} \left(\frac{1}{kL_1}\right)^4 \left(\frac{L_1}{z}\right)^{-2c}.
\end{aligned} \tag{5.21}
$$

Neglecting the $\mathtt{t}_R$ contribution and following the same procedure as in the flat case, we find

$$
\epsilon_3 = \int_{L_0}^{L_1} dz h_Y(0,z) \left\{ \frac{1}{kz} \frac{\tilde{g}^2}{g_5^2} h_W(0,z) - b_L(z) \right\}. \tag{5.22}
$$

Hence the ideal cancellation for this parameter is obtained when

$$
h_W(0,z) = \left(\frac{g_5^2}{\tilde{g}^2} \right) kz b_L(z). \tag{5.23}
$$

By considering the behaviour in z of $b_L(z)$ given in eq. (5.21), and that of $h_W(0,z)$ given in eq. (5.11), from the condition eq. (5.23) we may argue that the ideally delocalized left-handed fermions could be obtained with the choice $c = -\frac{1}{2}$, [34]. Nevertheless, because of the constant term in h_W, to satisfy exactly the condition (5.23) we must require $L_1 = 0$ and $L_0 = (\mathtt{t}_L^2 g_5^2/(\tilde{g}^2 \hat{g}_5^2))^{1/3}/k$. This means an inversion of the branes and a singular metric on $z = L_1 = 0$ because of the curvature factor $\frac{1}{kz}$, [49]. So, also in the case of RS warped metric, it is not possible to link the delocalization functions of the gauge boson and of the left-handed fermion in such a way to satisfy the bulk equations of motion and the ideal cancellation request.

The possibility of a global cancellation between the gauge and fermion contributions to ϵ_3 is obviously viable also for the warped metric case [34–36, 49].

6. Conclusions

The holographic prescription applied to the five dimensional Dirac theory, as well as to the five dimensional Yang-Mills theory, offers an alternative approach to the deconstruction analysis of the Higgsless models for studying low energy effective Lagrangians. The holographic technique used here is equivalent to the elimination of the fields of the internal sites of the moose in terms of the light fields $\tilde{W}$ and $\tilde{Y}$ [36, 49]. This last procedure can generate also current-current interactions in the low energy Lagrangian. These terms are absent in the simplest holographic analysis since one solves the free equations of motions for the bulk field. However following [39] it could be possible to reproduce also the current-current interaction terms with a suitable perturbative approach. The aforementioned equivalence has been shown in this paper, neglecting current-current terms, by studying a minimal Higgsless model based on the symmetry SU(2) broken by boundary conditions in the limit $\tilde{g}^2 \pi R / g_5^2 \ll 1$ which corresponds in the deconstructed theory to the limit $\tilde{g}^2/g_i^2 \ll 1$ where g_i is the coupling constant of the gauge group of the i-th site.

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In particular we have shown that though an ideal delocalization of the fermions along the extra dimension is not allowed by the bulk equations of motion, whatever the metric, a global cancellation of the ϵ_3 parameter is possible, and therefore the electroweak constraints can be satisfied. In the 5D formulation of the model there is still an interaction which appears to be non local in the fifth dimension. This non-locality could be eliminated by extending the 5D symmetry to $\mathrm{SU}(2) \times \mathrm{U}(1)_{B-L}$ and asking for suitable boundary conditions.

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A. Some useful identities

Let us write the bulk chiral fermions in terms of two degrees of freedom q_L and q_R with left and right chirality respectively as

$$\begin{aligned} \psi_L(p,y) &= f_L(p,y)\mathtt{t}_L q_L(p) + \not{p}\pi R\tilde{f}_L(p,y)\mathtt{t}_R q_R(p), \\ \psi_R(p,y) &= f_R(p,y)\mathtt{t}_R q_R(p) + \not{p}\pi R\tilde{f}_R(p,y)\mathtt{t}_L q_L(p)\,. \end{aligned} \tag{A.1}$$

Taking into account eqs. (3.3), the functions $f_{L,R}$ and $\tilde{f}_{L,R}$ satisfy the following differential equations

$$\begin{aligned} &\mathtt{t}_L f_L + \pi R(\partial_5 - M)\mathtt{t}_R\tilde{f}_R = 0, && p^2\pi R\mathtt{t}_R\tilde{f}_R - (\partial_5 + M)\mathtt{t}_L f_L = 0, \\ &\mathtt{t}_R f_R - \pi R(\partial_5 + M)\mathtt{t}_L\tilde{f}_L = 0, && p^2\pi R\mathtt{t}_L\tilde{f}_L + (\partial_5 - M)\mathtt{t}_R f_R = 0, \end{aligned} \tag{A.2}$$

which can be decoupled as

$$f''_{L,R} + \omega^2 f_{L,R} = 0, \quad \tilde{f}''_{L,R} + \omega^2\tilde{f}_{L,R} = 0\,, \tag{A.3}$$

in analogy with eq. (3.4).

Furthermore multiplying the first of eqs. (A.2) by f_L, integrating over y, and using the third of eq. (A.2) we get the following useful identity

$$\int_0^{\pi R} dy f_L^2(p,y) = -\pi R[f_L(p,y)\tilde{f}_R(p,y)]_0^{\pi R} + (\pi R)^2 p^2 \int_0^{\pi R} dy\tilde{f}_R^2(p,y)\,. \tag{A.4}$$

In analogous way by multiplying the second of eqs. (A.2) by f_R, integrating over y, and using the forth of eq. (A.2) we get

$$\int_0^{\pi R} dy f_R^2(p,y) = \pi R[f_R(p,y)\tilde{f}_L(p,y)]_0^{\pi R} + (\pi R)^2 p^2 \int_0^{\pi R} dy\tilde{f}_L^2(p,y)\,, \tag{A.5}$$

By evaluating eq. (A.4)–(A.5) at $p = 0$, taking into account the boundary conditions eqs. (3.7) which imply that $f_L(p,0) = f_R(p,\pi R) = 1$, we get

$$\begin{aligned} \int_0^{\pi R} dy f_L^2(0,y) &= -\pi R[f_L(0,y)\tilde{f}_R(0,y)]_0^{\pi R} = \pi R\tilde{f}_R(0,0), \\ \int_0^{\pi R} dy f_R^2(0,y) &= \pi R[f_R(0,y)\tilde{f}_L(0,y)]_0^{\pi R} = \pi R\tilde{f}_L(0,\pi R)\,. \end{aligned} \tag{A.6}$$

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